

Chapter 4 / **Example 18****Sum of an arithmetic series**

&lt;A short introductory text, where appropriate&gt;

- a** Find the sum of the arithmetic series  $-10 + (-6) + (-2) + \dots + 90$ .  
**b** Write down this series in sigma notation.  
**c** Find the least number of terms from this series needed to obtain a sum greater than 100.

$$u_1 = -10 \text{ and } d = 4$$

$$u_n = -10 + 4(n-1)$$

Press  $[F1]$   $[Y=]$  to display the equation entry screen.

Type  $-10 + 4(x-1)$  and press  $[ENTER]$ .

Plot1 Plot2 Plot3  
 $\text{Y}_1 = -10 + 4(X-1)$   
 $\text{Y}_2 =$   
 $\text{Y}_3 =$   
 $\text{Y}_4 =$   
 $\text{Y}_5 =$   
 $\text{Y}_6 =$   
 $\text{Y}_7 =$   
 $\text{Y}_8 =$   
 $\text{Y}_9 =$

Press  $[2nd]$   $[F5]$  ( $[TABLE]$ ).

You can scroll down the table using  $\downarrow$ .

From the table,  $u_n = 90$  when  $n = 26$

$$S_{26} = \frac{26}{2}(-10 + 90) = 1040$$

X	Y <sub>1</sub>				
18	58				
19	62				
20	66				
21	70				
22	74				
23	78				
24	82				
25	86				
26	90				
27	94				
28	98				

$Y_1 = 90$

Press  $[F1]$   $[Y=]$  to display the equation entry screen.

Press  $[ALPHA]$   $[F2]$  2:summation  $\Sigma($

The template has spaces for the variable,  $i$ , the limits and the function.

Plot1 Plot2 Plot3  
 $\text{Y}_1 = -10 + 4(X-1)$   
 $\text{Y}_2 = \sum_{i=1}^X (-10 + 4(I-1))$   
 $\text{Y}_3 =$   
 $\text{Y}_4 =$   
 $\text{Y}_5 =$   
 $\text{Y}_6 =$   
 $\text{Y}_7 =$   
 $\text{Y}_8 =$

Type  $I = 1$ ,  $X$  and  $-10 + 4(I-1)$  in the template as  $Y_2$ .

Press  $[ENTER]$ .

Plot1 Plot2 Plot3  
 $\text{Y}_1 = -10 + 4(X-1)$   
 $\text{Y}_2 = \sum_{I=1}^X (-10 + 4(I-1))$   
 $\text{Y}_3 =$   
 $\text{Y}_4 =$   
 $\text{Y}_5 =$   
 $\text{Y}_6 =$   
 $\text{Y}_7 =$   
 $\text{Y}_8 =$

Chapter 4 / **Example 18****Sum of an arithmetic series**

Press **[2nd]** **[F5]** (**[TABLE]**).

You can scroll down the table using **[↓]**.

The sum is smaller than 100 when  $n = 10$ , and larger when  $n = 11$ .

The the least number of terms is 11.

X	Y <sub>1</sub>	Y <sub>2</sub>			
3	-2	-18			
4	2	-16			
5	6	-10			
6	10	0			
7	14	14			
8	18	32			
9	22	54			
10	26	80			
11	30	110			
12	34	144			
13	38	182			

Y<sub>2</sub>=110

Alternatively, the sum can be expressed as

$$S_n = \frac{n}{2}(2(-10) + 4(n-1))$$

Press **[F1]** **[Y=]** to display the equation entry screen.

Type  $\frac{x}{2}(2(-10) + 4(x-1))$  and press **[ENTER]** to enter the function as Y<sub>3</sub>.

You can deselect Y<sub>1</sub> and Y<sub>2</sub> by scrolling up to the = alongside the function and pressing **[ENTER]**.

Plot1	Plot2	Plot3
Y <sub>1</sub> = -10+4(X-1)		
Y <sub>2</sub> = $\sum_{I=1}^X (-10+4(I-1))$		
Y <sub>3</sub> = $\frac{X}{2}(2(-10)+4(X-1))$		
Y <sub>4</sub> =		
Y <sub>5</sub> =		
Y <sub>6</sub> =		
Y <sub>7</sub> =		

To show the graph in an appropriate window.

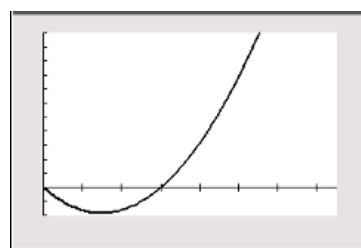
Press **[F2]** **[WINDOW]**

Set the axes to show  $0 \leq x \leq 15$  with a scale of 2 and  $-20 \leq y \leq 110$  with a scale of 10. Leave other items the same.

Press **[F5]** **[GRAPH]** when you have finished.

WINDOW
Xmin=0
Xmax=15
Xscl=2
Ymin=-20
Ymax=110
Yscl=10
Xres=1
ΔX=.05681818181818
TraceStep=.11363636363636

The GDC displays Y<sub>3</sub> with suitable axes.



Press **[F1]** **[Y=]** to display the equation entry screen.

Type 100 and press **[ENTER]** to enter the function as Y<sub>4</sub>.

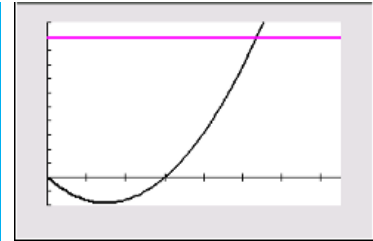
Plot1	Plot2	Plot3
Y <sub>1</sub> = -10+4(X-1)		
Y <sub>2</sub> = $\sum_{I=1}^X (-10+4(I-1))$		
Y <sub>3</sub> = $\frac{X}{2}(2(-10)+4(X-1))$		
Y <sub>4</sub> = 100		
Y <sub>5</sub> =		
Y <sub>6</sub> =		
Y <sub>7</sub> =		

Chapter 4 / **Example 18****Sum of an arithmetic series**

The GDC now displays the curve and the straight-line graph:

$$Y_3 = \frac{x}{2}(2(-10) + 4(x-1))$$

$$Y_4 = 100$$

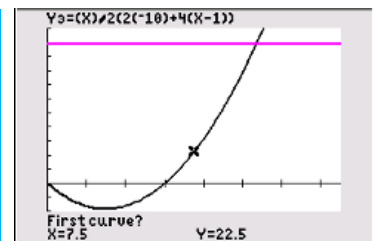


Press **[2nd]** **[F4]** **[CALC]** 5:intersect

To find the intersection you need to choose the two lines that intersect.

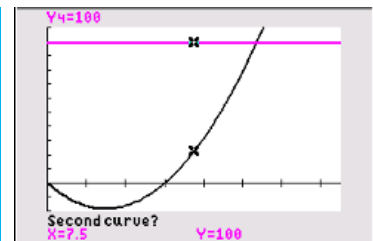
The GDC shows a cross on one of the lines and 'First curve?'.

Press **[ENTER]**.



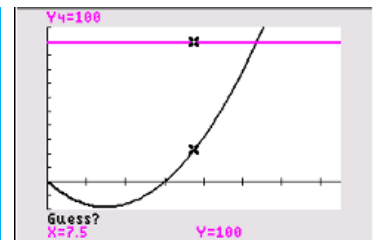
The GDC shows a cross on the other line and 'Second curve?'.

Press **[ENTER]**.



The GDC requires an initial guess for the position of the intersection. Choose the default position.

Press **[ENTER]**.



The GDC displays the intersection of the two straight lines at the point (10.7,100)

As  $n$  must be a whole number and the sum must be greater than 100, you choose the next largest whole number, so  $n = 11$ .

